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Solving Parabolic Interface problems with a Finite Element Method

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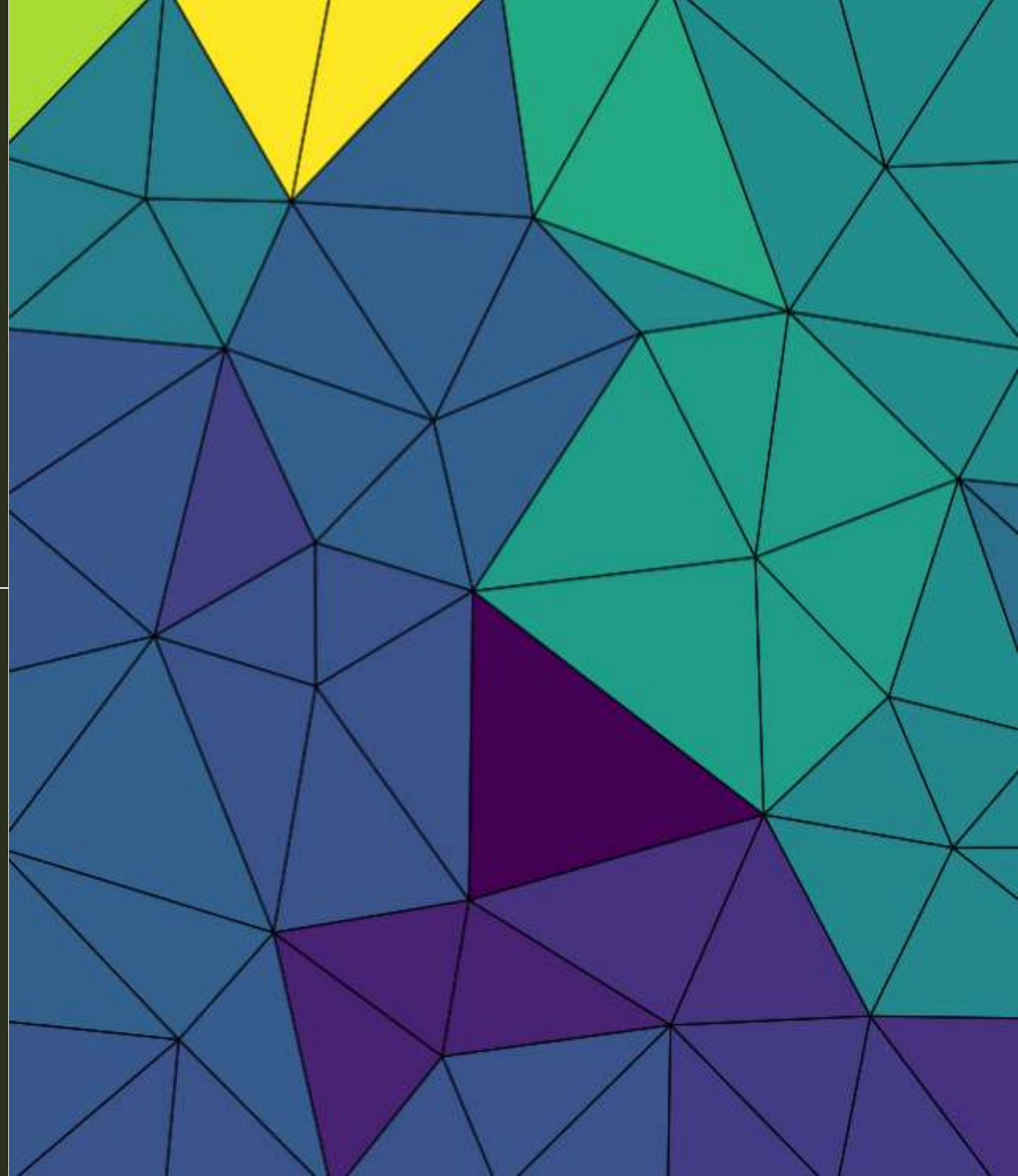
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Solving Parabolic Interface Problems with a Finite Element Method

Henry Brown

WCU Research & Creative Activity Day
4-29-2021



Parabolic Interface PDEs

- * Defined on a domain which is split by an interface.
- * Solutions may be discontinuous over the interface.

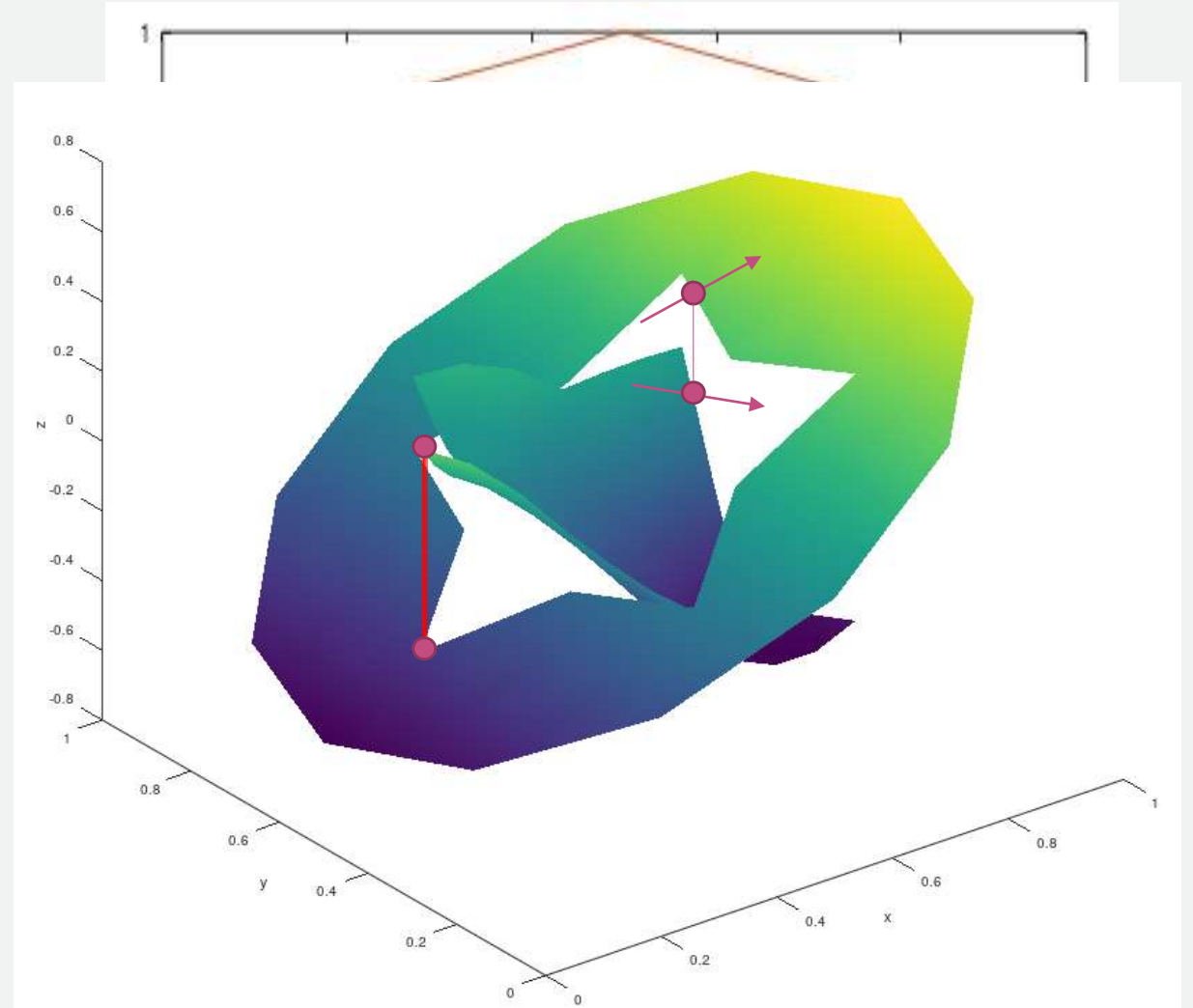
$$\frac{\partial u}{\partial t} = \nabla \cdot (\alpha \nabla u) + f, \quad x \in \Omega$$

$$u = g_D, \quad x \in \partial\Omega_D$$

$$\alpha \frac{\partial u}{\partial n} = g_N, \quad x \in \partial\Omega_N$$

$$[u] = u^+ - u^- = \phi, \quad x \in \Gamma$$

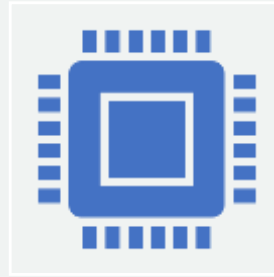
$$[\alpha \frac{\partial u}{\partial n}] = \alpha^+ \frac{\partial u^+}{\partial n} - \alpha^- \frac{\partial u^-}{\partial n} = \psi, \quad x \in \Gamma$$



Why Computational Methods?



Difficulty/impossibility of finding closed form analytical solutions



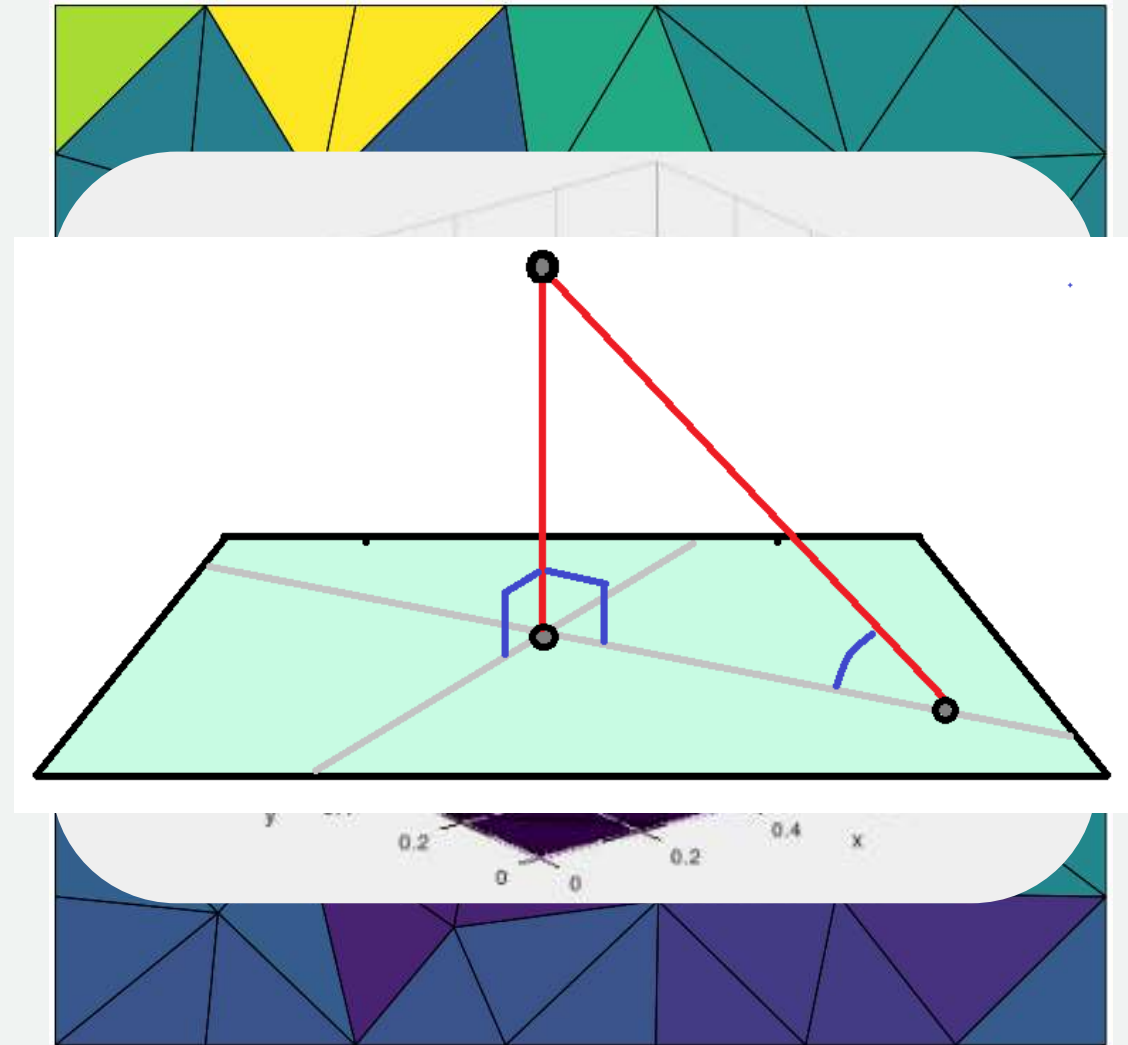
Handle a varying array problems without added complexity



These methods have proven error bounds and convergence

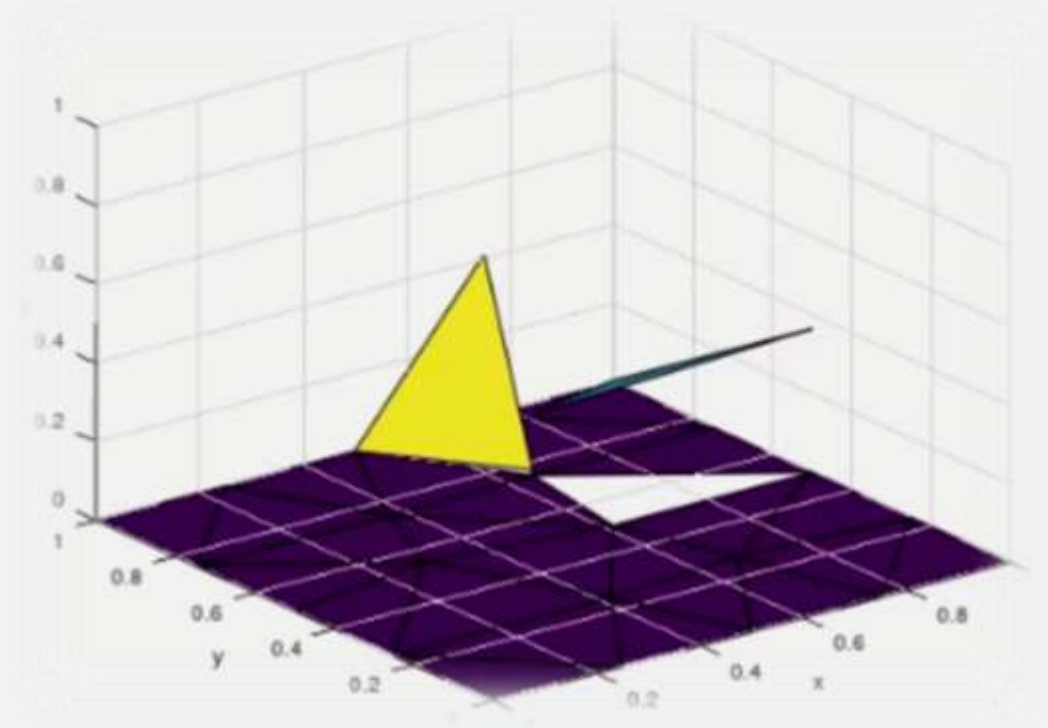
Finite Element Methods (FEMs)

- * Can use non-uniform meshes
- * Piecewise polynomials over the elements using basis functions
- * Projection theorems ensure that FEM finds the best approximation in the function space



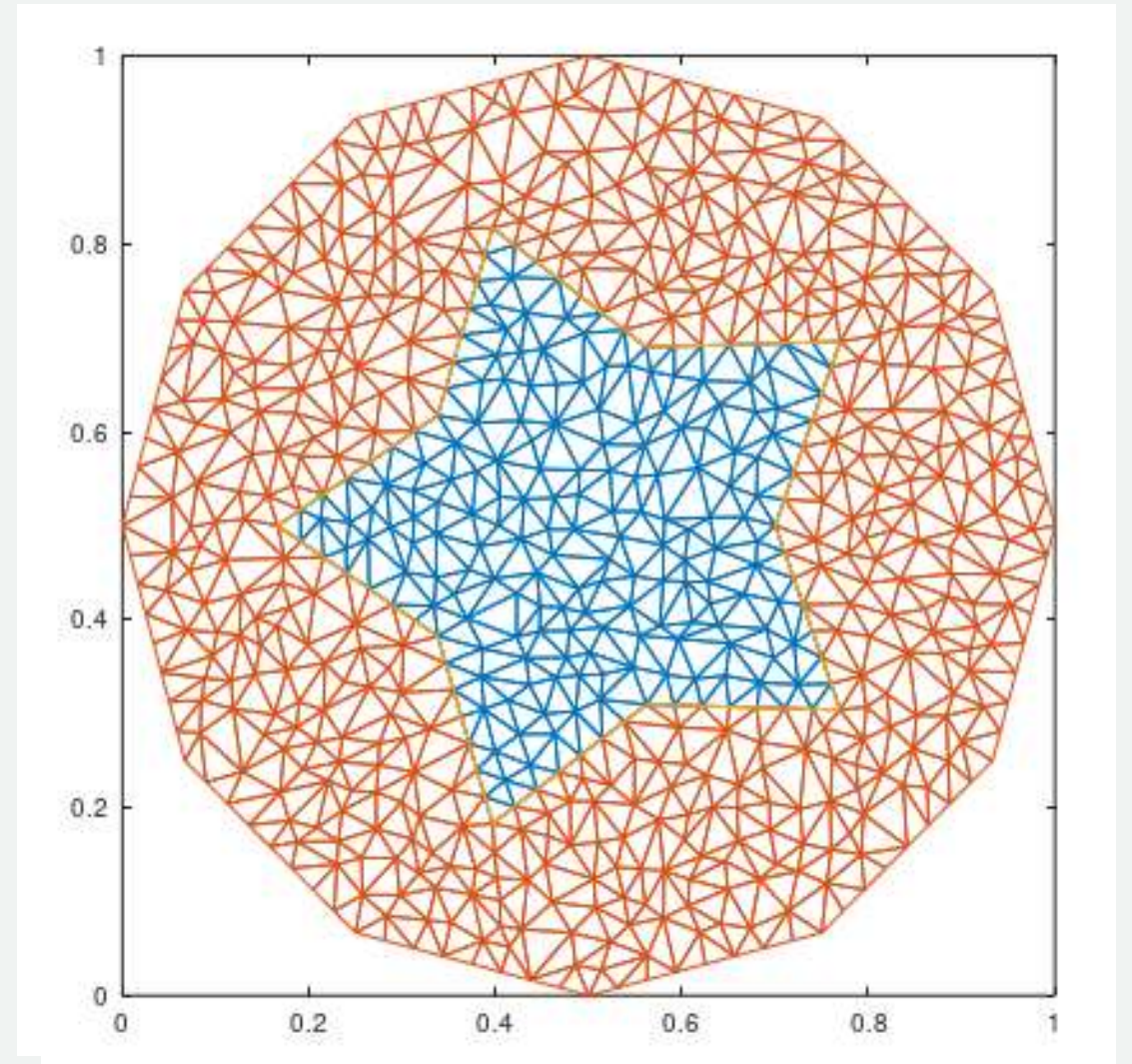
Discontinuous Galerkin (DG) FEMs

- * Allows for discontinuity over element boundaries
- * Broken piecewise polynomials over the elements
- * A natural means to apply the jump conditions in the parabolic interface problems



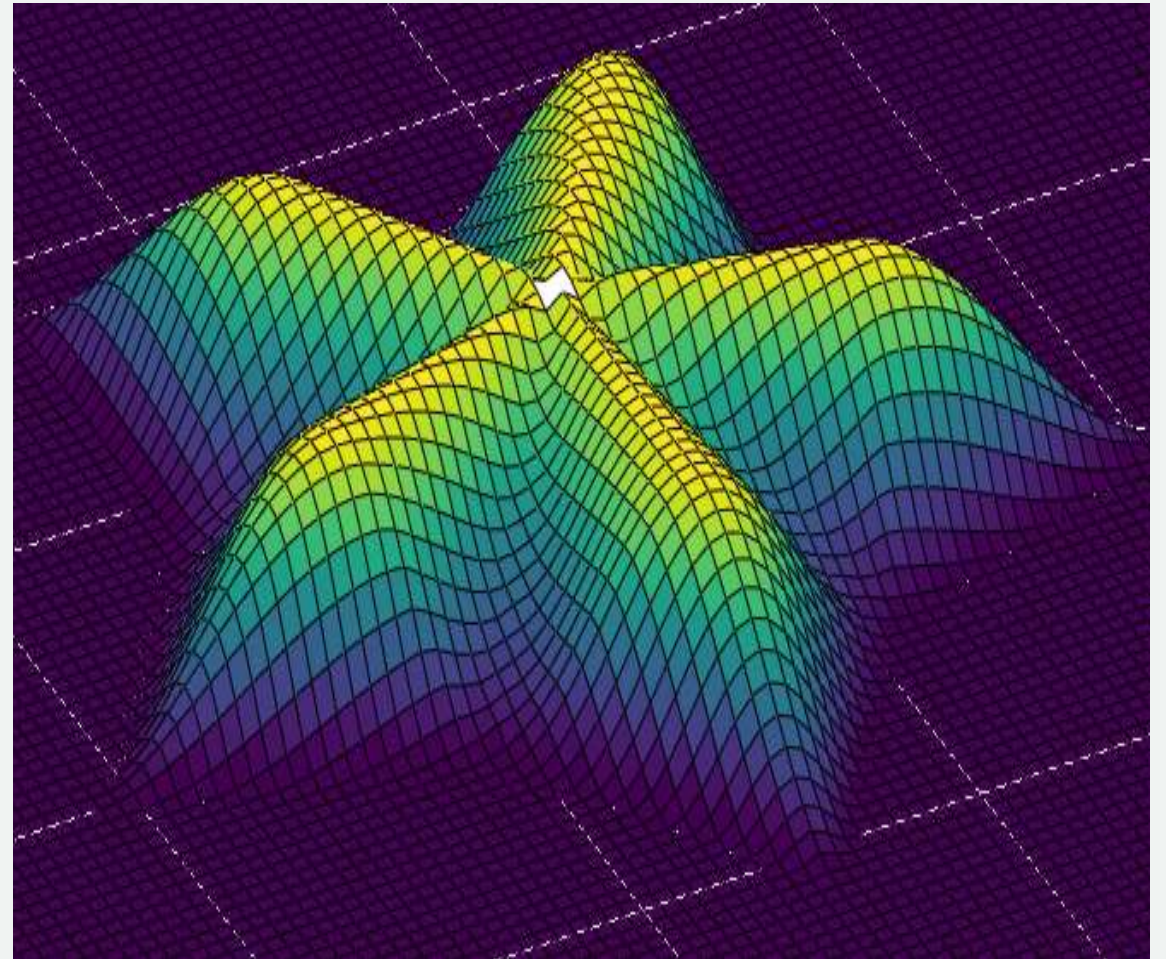
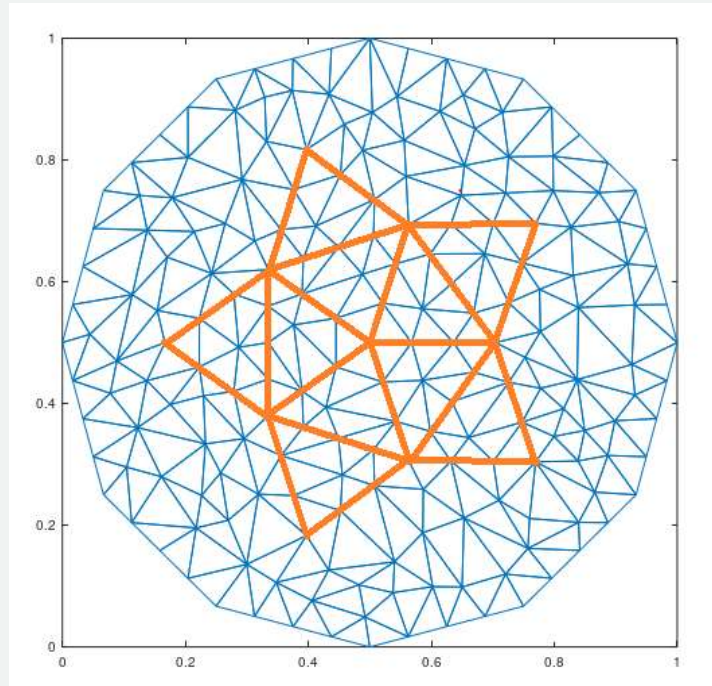
Creating a Conforming Triangulation

- * Triangulation which conforms to the interface
- * Triangulation can be refined, which gives room more accurate solution



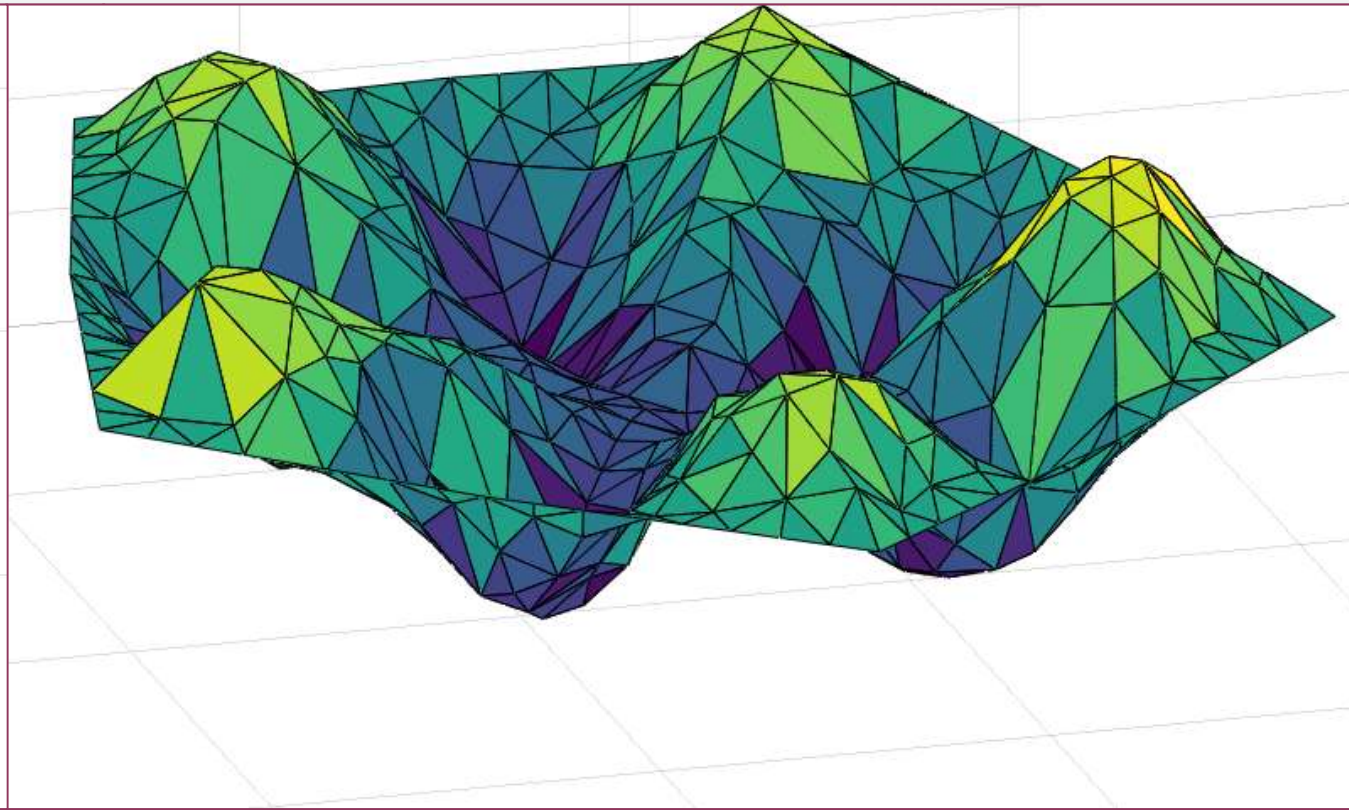
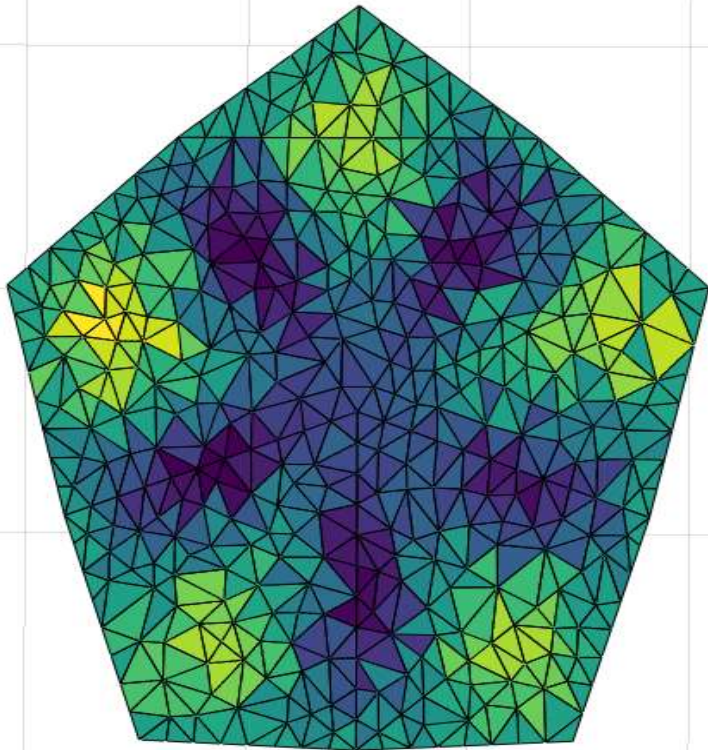
First Test Problem (Starfish)

- * Zero if not in the star
- * Piecewise source term
- * Initial Condition (Right)



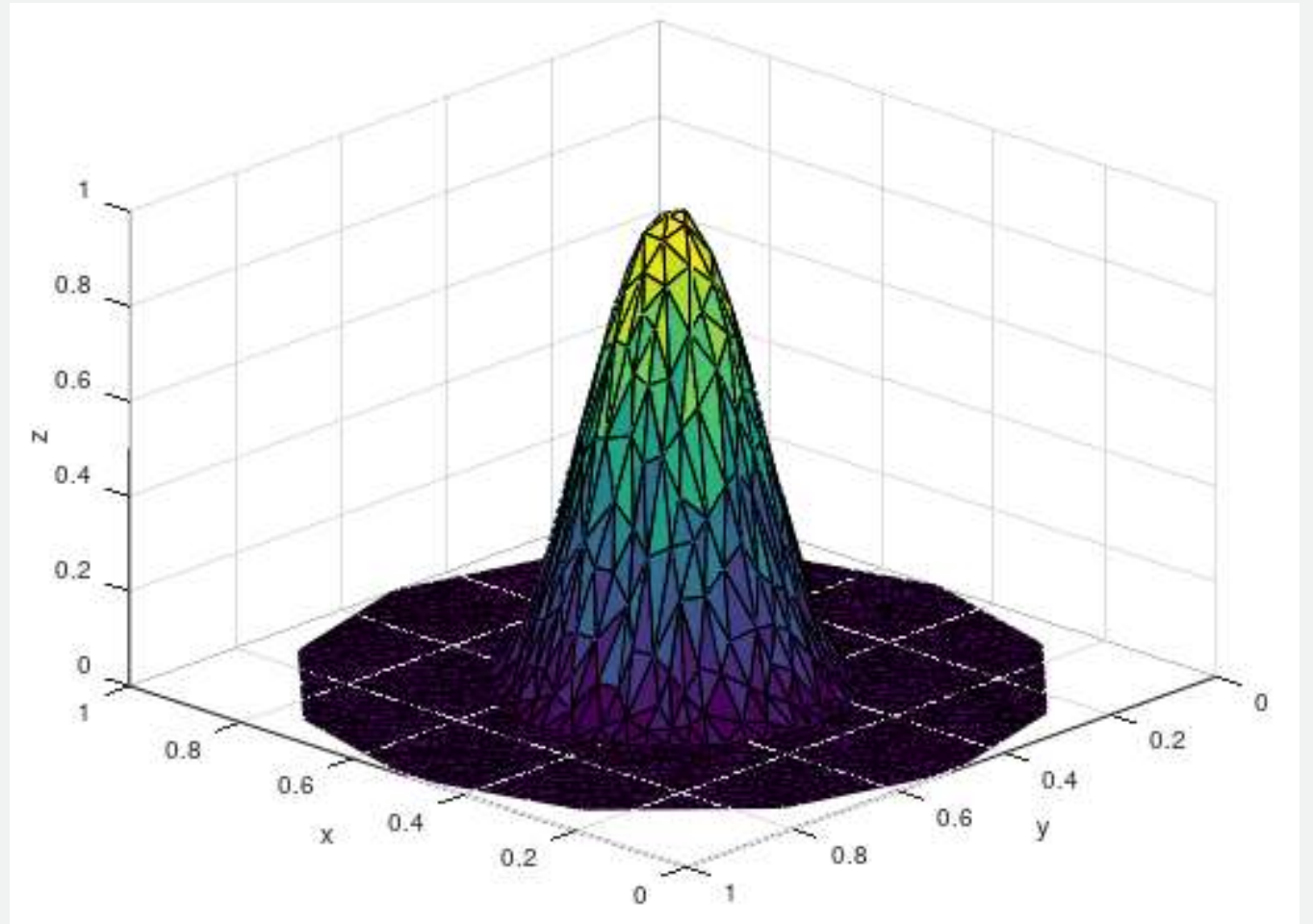
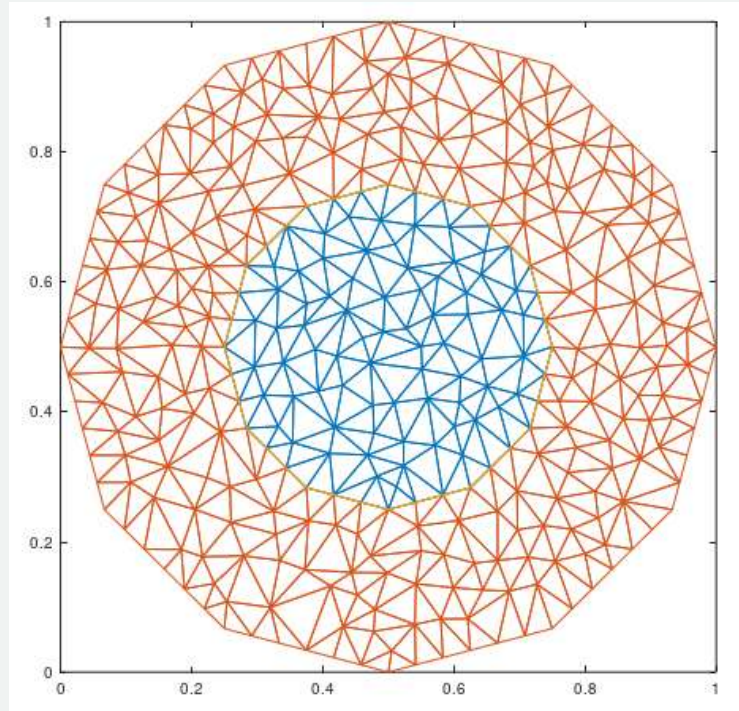
Issues with the Test Problem

- * Sinking behavior
- * Trouble with the center

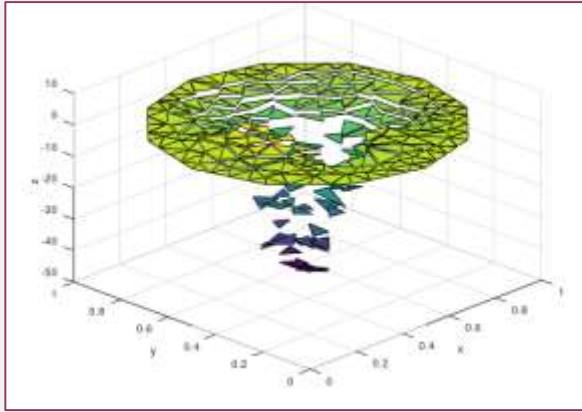


Changing the Test Problem

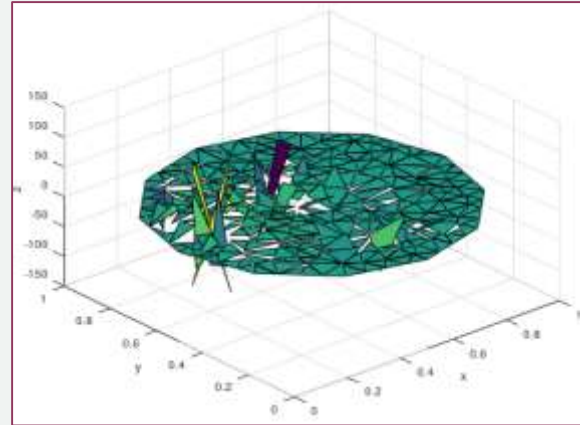
- * Less complex geometry
- * Initial Condition (Right)



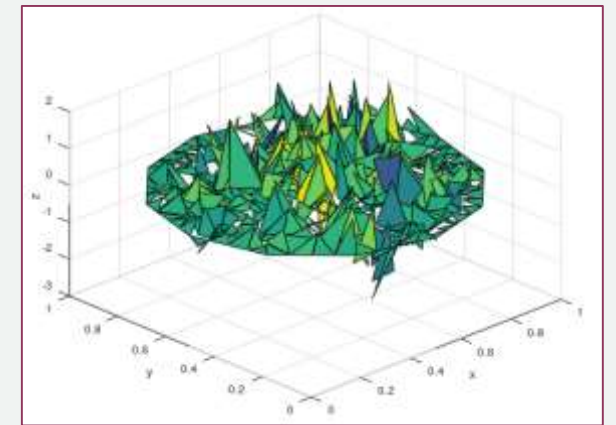
Stability and Over-Penalizing the Interface



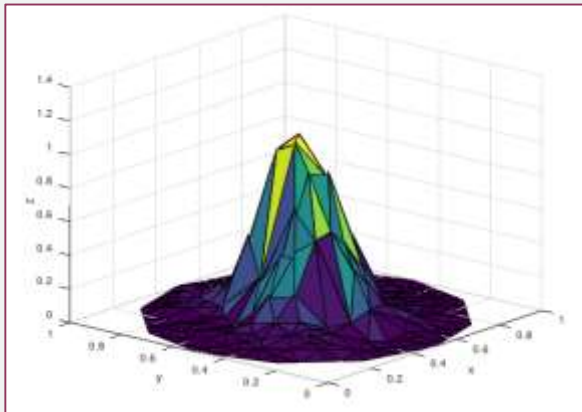
Penalty: 800



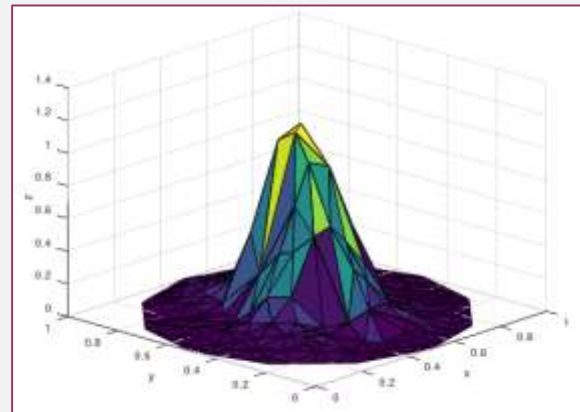
Penalty: 8000



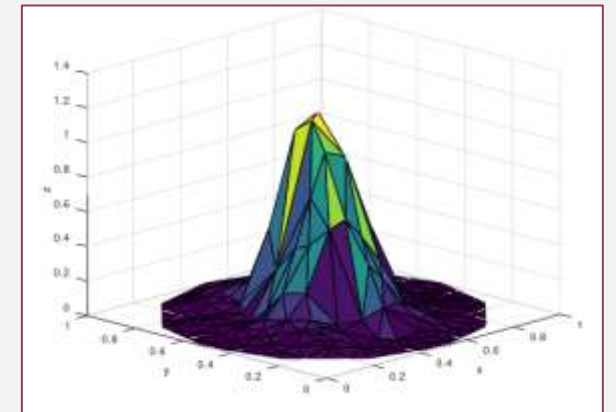
Penalty: 80000



Penalty: 800000

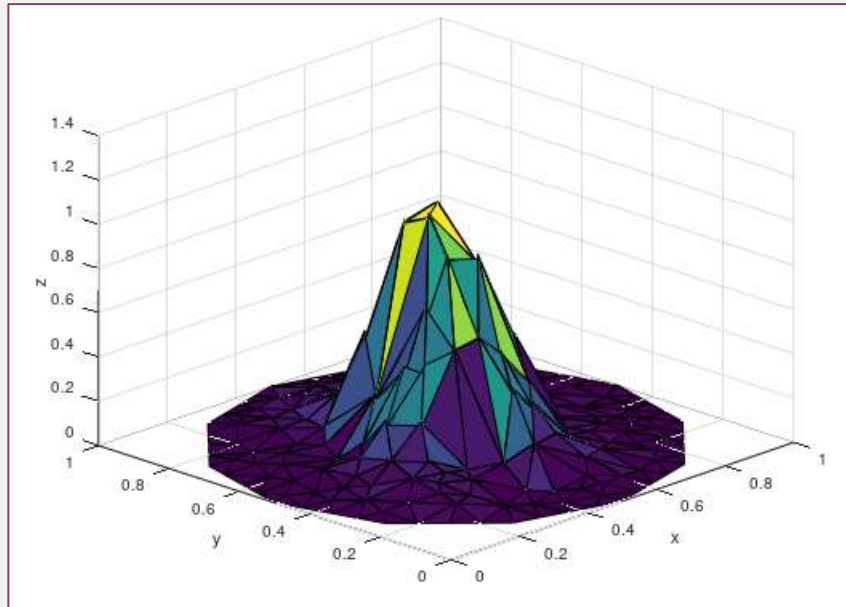


Penalty: 8000000

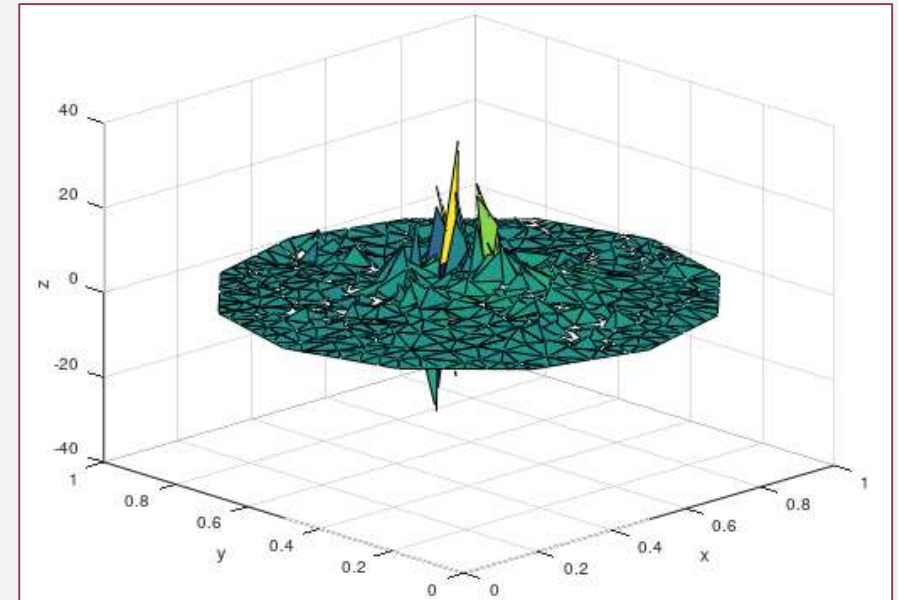
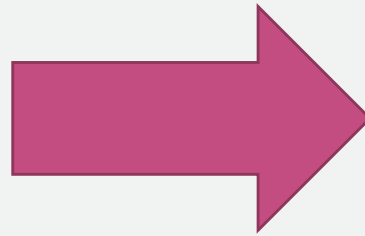


Penalty: 8×10^{13}

Penalty Dependence on Mesh



Density: 300

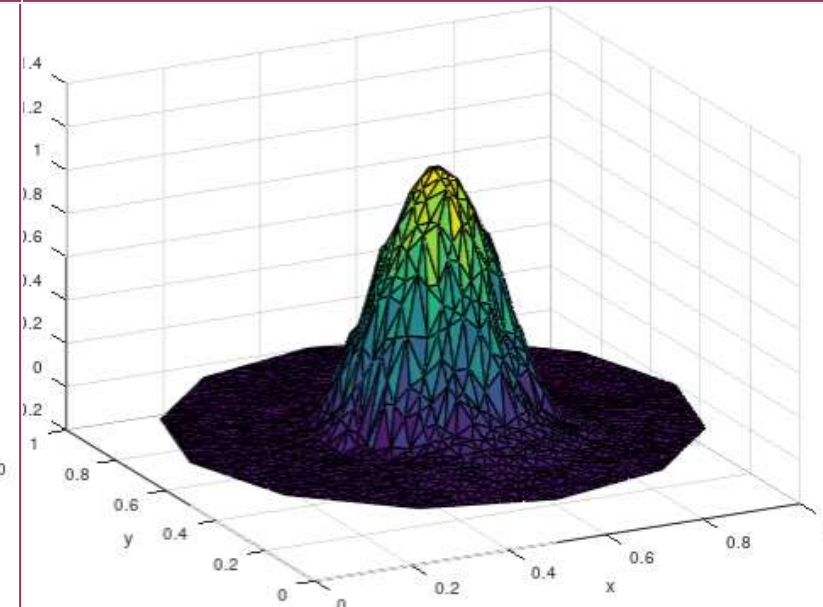
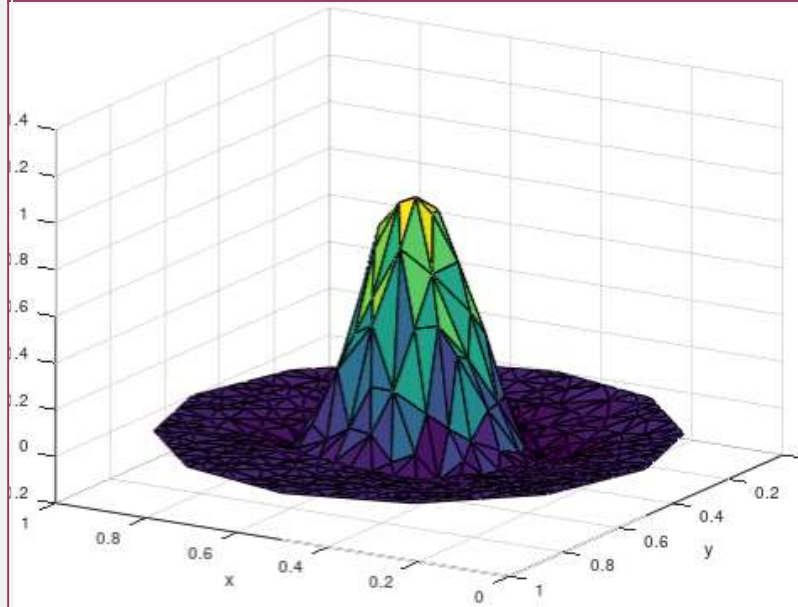
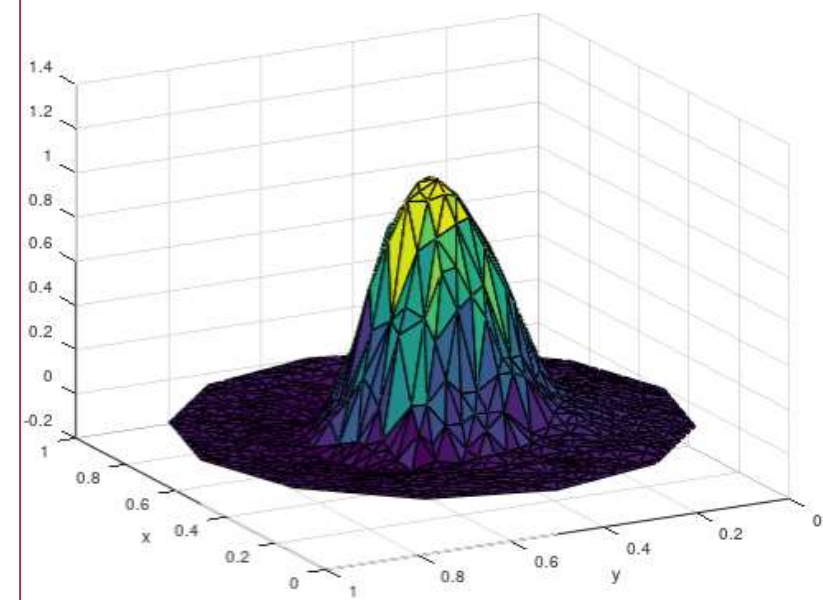
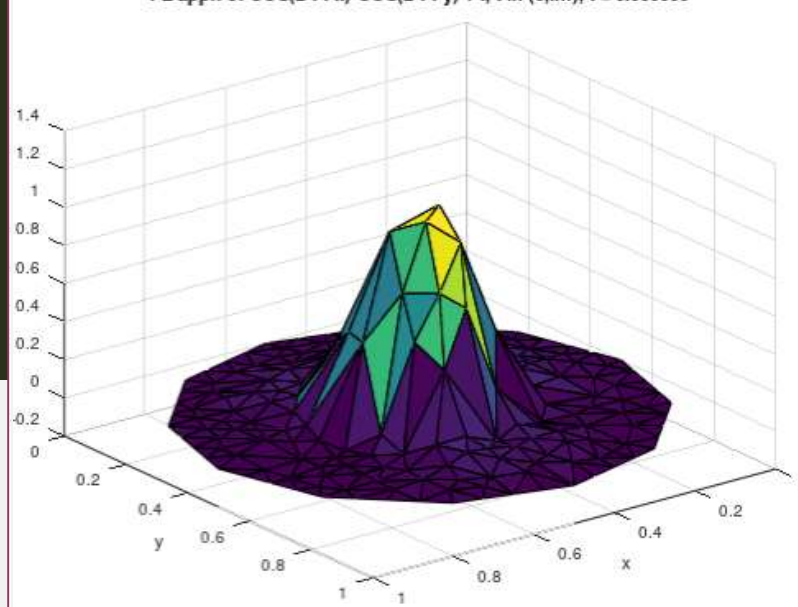


Density: 600

- * Penalty: 800000
- * Optimally, no dependence on mesh

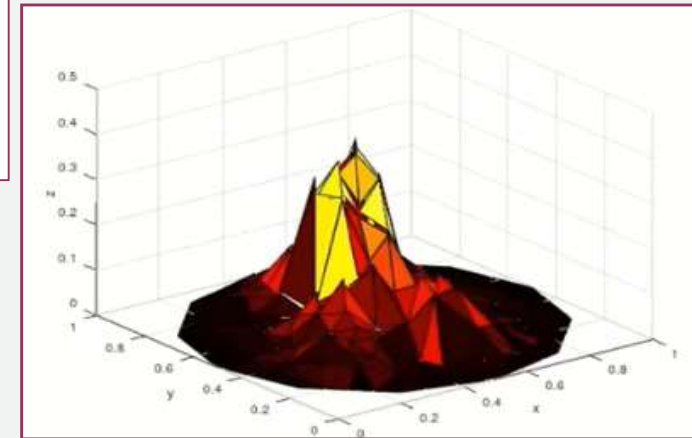
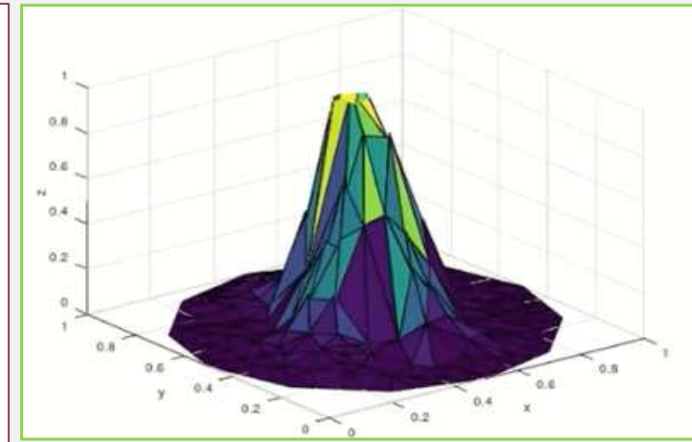
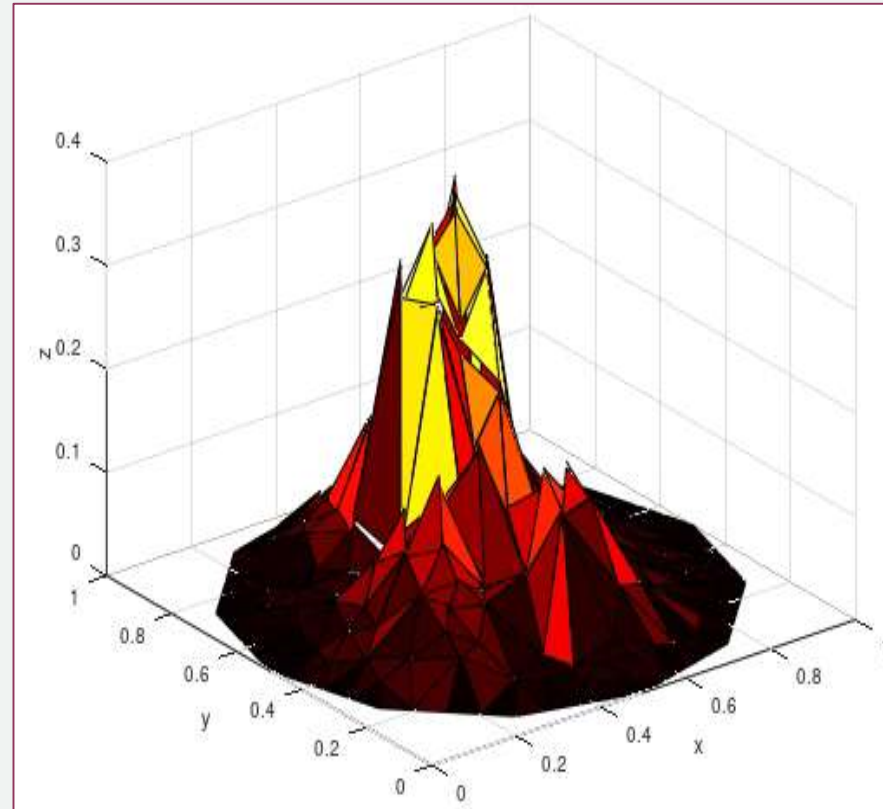
Elliptic Problem Over Mesh Refinement

- * Keeps shape of exact solution
- * Error around interface does not diminish
- * Not optimal with Over-penalizing



Solving the Interfaced Heat Equation

- * Sufficiently small time step
- * Oscillation for larger time steps
- * Stable in time without over-penalizing



Approximation and Error Plots

Discussion

- * Method works for interface conditions equal to zero
- * No success with non-zero interface conditions
- * Over-penalizing required due to triangulation orientation
- * Non-zero interface conditions (WG FEM, DDG FEM)

Acknowledgements



- * Dr. Andreas Aristotelous
- * Dr. Chuan Li

Questions?